UNIT 6: MENSURATION

CHAPTER 12

AREAS RELATED TO CIRCLES

Syllabus

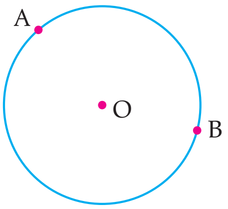
- **Motivate the area of a circle; area of the sectors and the segments of a circle. Problems based on areas and perimeter/circumference of the plane figures of circles. (In calculating area of segment of a circle, problems should be restricted to central angle of 60°, 90° and 120° only. Plane figures involving triangles, simple quadrilaterals and circle.)**

Trend Analysis

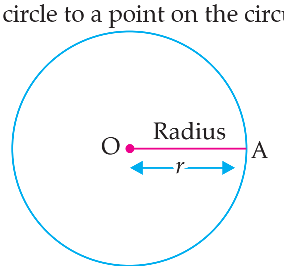
List of Concepts	2018		2019		2020	
	Delhi	Outside Delhi	Delhi	Outside Delhi	Delhi	Outside Delhi
Area of Shaded Region	1 Q (3 M)		1 Q (3 M)	2 Q (3 M)	3 Q (2 M)	2 Q (1 M) 1 Q (3 M) 1 Q (4 M)

Revision Notes

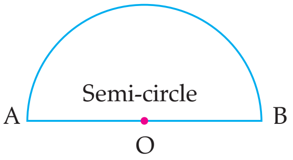
- A circle is a collection of all points in a plane which are at a constant distance from a fixed point in the same plane.

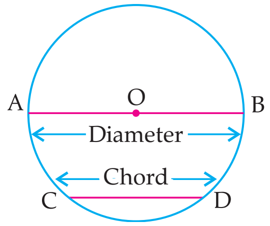


- A line segment joining the centre of the circle to a point on the circumference of the circle is called its radius.

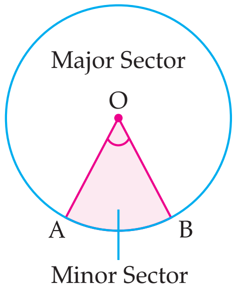


- A line segment joining any two points of a circle is called a chord. A chord passing through the centre of circle is called its diameter. A diameter is the largest chord of the circle. Here AB is a a diameter, which is a longest chord.
- A diameter of a circle divides a circle into two equal arcs, each known as a semi-circle.

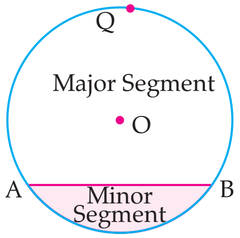




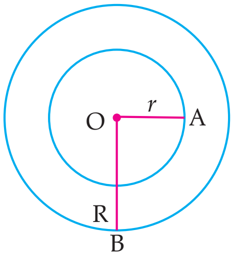
- A part of a circumference of circle is called an arc.
- An arc of a circle whose length is less than that of a semi-circle of the same circle is called a minor arc.
- An arc of a circle whose length is greater than that of a semi-circle of the same circle is called a major arc.
- The region bounded by an arc of a circle and two radii at its end points is called a sector.



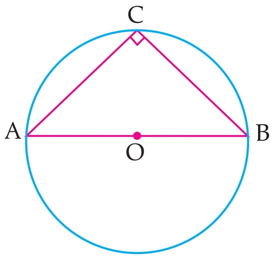
- A chord divides the interior of a circle into two parts, each called a segment.



- Circles having the same centre but different radii are called concentric circles.



- Two circles (or arcs) are said to be congruent if on placing one over the other cover each other completely.
- The distance around the circle or the length of a circle is called its circumference or perimeter.
- The mid-point of the hypotenuse of a right triangle is equidistant from the vertices of the triangle.
- Angle subtended at the circumference by a diameter is always a right angle.



- Angle described by minute hand in 60 minutes is 360°.
- Angle described by hour hand in 12 hours is 360°.

Know the Formulae

1. Circumference (perimeter) of a circle = πd or $2\pi r$, where d is diameter and r is the radius of the circle.
2. Area of a circle = πr^2.
3. Area of a semi-circle = $\frac{1}{2}\pi r^2$.
4. Perimeter of a semi-circle = $\pi r + 2r = (\pi + 2)r$
5. Area of a ring or an annulus = $\pi(R + r)(R - r)$. where R is the outer radius and r is the inner radius.
6. Length of arc, $l = \frac{2\pi r\theta}{360°}$ or $\frac{\pi r\theta}{180°}$, where θ is the angle subtended at centre by the arc.
7. Area of a sector = $\frac{\pi r^2\theta}{360°}$

   or area of sector = $\frac{1}{2}(l \times r)$, where l is the length of arc.
8. Area of minor segment = $\frac{\pi r^2\theta}{360°} - \frac{1}{2}r^2 \sin\theta$.
9. Area of major segment = Area of the circle – Area of minor segment

   = πr^2 – Area of minor segment.
10. If a chord subtends a right angle at the centre, then

    area of the corresponding segment = $\left[\frac{\pi}{4} - \frac{1}{2}\right]r^2$
11. If a chord subtends an angle of 60° at the centre, then

    area of the corresponding segment = $\left(\frac{\pi}{6} - \frac{\sqrt{3}}{4}\right)r^2$.
12. If a chord subtends an angle of 120° at the centre, then

    area of the corresponding segment = $\left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right)r^2$.
13. Distance moved by a wheel in 1 revolution = Circumference of the wheel.
14. Number of revolutions in one minute = $\frac{\text{Distance moved in 1 minute}}{\text{Circumference}}$.
15. Perimeter of a sector = $\frac{\pi r\theta}{180°} + 2r$.

Know the Facts

- An Indian mathematician Srinivas Ramanujan worked out the identity using the value of π correct to million places of decimals.
- The Indian mathematician Aryabhatta gave the value of π as $\frac{62832}{20000}$
- "How I made a greater discovery" this mnemonic help us in getting the value of π = 3.14159 ....
- Give it under separate reading with explanation how to use

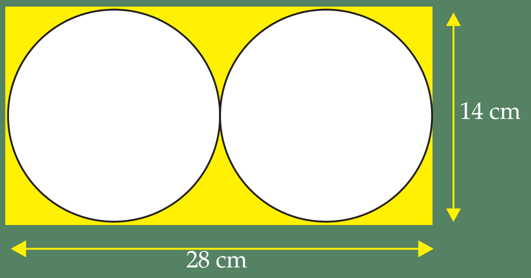
	CAN	I	HAVE	A	SMALL	CONTAINER	OF	COFFEE
	↓	↓	↓	↓	↓	↓	↓	↓
No. of Letters →	3	1	4	1	5	9	2	6

- Archimedes calculated the area of a circle by approximating it to a square.
- Area of sector of a circle depends on two parameters-radius and central angle.

How is it done on the GREENBOARD?

Q.1. Find the area of the shaded region in the given figure.

[Delhi - 2013] U



Solution:

Step I: Length of rectangle = 28 cm

Breadth of rectangle = 14 cm

∴ Diameter of circle = 14 cm

or, radius of circle = 7 cm

Step II: Area of rectangle

$= 28 \times 14 \text{ cm}^2$

$= 392 \text{ cm}^2$

Area of each circle $= \frac{22}{7} \times (7)^2$

$= 154 \text{ cm}^2$

Area of both circles $= 2 \times 154 \text{ cm}^2$

$= 308 \text{ cm}^2$

Step III: Shaded area = Area of rectangle – Area of both circles

$= 392 - 308$

$= 84 \text{ cm}^2$

Very Short Answer Type Questions — 1 mark each

AI **Q. 1. In a circle of diameter 42 cm, if an arc subtends an angle of 60° at the centre, where $\pi = \frac{22}{7}$, then what will be the length of arc.**

U [CBSE SQP, 2020-21]

Sol. Length of arc $= \frac{\theta}{360°}(2\pi r)$ ½

$= \frac{60°}{360°}\left(2 \times \frac{22}{7} \times 21\right)$

$= 22$ cm ½

[CBSE Marking Scheme, 2020-21]

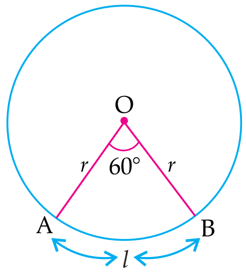
Detailed Solution:

Given, diameter of the circle = 42 cm

Then, radius of the circle $= \frac{42}{2}$ cm = 21 cm.

and angle subtended at the centre,

$\theta = 60°$



½

∴ The length of arc, $l = \frac{\theta}{360°} \times 2\pi r$

$= \frac{60°}{360°} \times 2 \times \frac{22}{7} \times 21$

$= 22$ cm ½

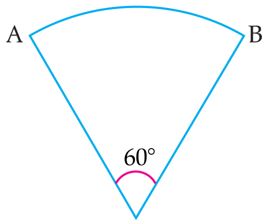
COMMONLY MADE ERROR

- Some students used incorrect formula for length of arc and some made mistakes in calculation.

ANSWERING TIP

- Remember the formula of length of arc and the concept of angle subtended at the centre.

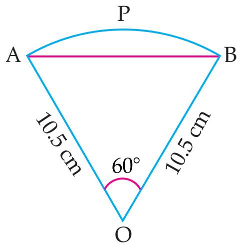
AI **Q. 2. In the given figure, find the perimeter of the sector of a circle with radius 10.5 cm and of angle 60°. $\left(\text{Take } \pi = \frac{22}{7}\right)$.**



U [CBSE OD Set-I, 2020]

Sol. We have, radius (r) = 10.5 cm

and angle (θ) = 60°



Then, the length of arc APB

$$= \frac{\theta}{360°} \times 2\pi r$$

$$= \frac{60°}{360°} \times 2 \times \frac{22}{7} \times 10.5$$

$$= 11 \text{ cm} \qquad \frac{1}{2}$$

Now, the perimeter of the sector OAPBO

$= OA$ + length of an arc $APB + BO$

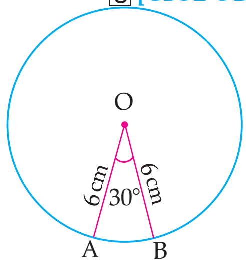
$= (10.5 + 11 + 10.5)$ cm

$= 32$ cm. ½

AI **Q. 3. Find the area of the sector of a circle of radius 6 cm whose central angle is 30°. (Take π = 3.14)**

U [CBSE OD Set-III, 2020]

Sol.



Given, radius of a circle,

$OA = OB = 6$ cm

(Assuming in figure) ½

and central angle $\theta = \angle AOB = 30°$

By using formula,

area of the sector of a circle

$$= \frac{\theta}{360°} \times \pi r^2$$

$$= \frac{30°}{360°} \times 3.14 \times 6 \times 6$$

$$= 9.42 \text{ cm}^2 \qquad \frac{1}{2}$$

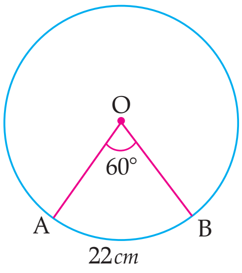
Short Answer Type Questions-I — 2 marks each

Q. 1. A piece of wire 22 cm long is bent into the form of an arc of a circle subtending an angle of 60° at its centre. Find the radius of the circle. $\left[\text{Use } \pi = \frac{22}{7}\right]$

A [CBSE Delhi Set-I, 2020]

Sol. AB is an arc of a circle.

Let radius of a circle be r cm.



i.e., $AB = 22$ cm

and $\theta = 60°$

$\therefore$ Length of arc $= \dfrac{2\pi r\theta}{360°}$ **1**

$$\Rightarrow \quad 22 = \frac{2 \times 22 \times r \times 60°}{7 \times 360°}$$

$$\Rightarrow \quad 22 = \frac{22 \times r}{21}$$

$$\Rightarrow \quad 22 \times r = 22 \times 21$$

$$\Rightarrow \quad r = 21$$

Hence, The radius of the circle (r) is 21 cm. **1**

Q. 2. The perimeter of a sector of a circle of radius 5.2 cm is 16.4 cm. Find the area of the sector.

A [CBSE Delhi Set-II, 2020]

Sol. Given, radius of circle (r) = 5.2 cm

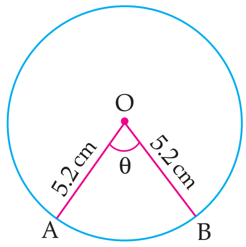
i.e., $OA = OB = r = 5.2$ cm

and the perimeter of a sector = 16.4 cm

As we know that perimeter of the sector

$$= 2r + \frac{2\pi r\theta}{360°}$$

$$\Rightarrow \quad 16.4 = 2 \times 5.2 + \frac{2\pi \times 5.2 \times \theta}{360°}$$



$$\Rightarrow \quad \frac{2\pi \times 5.2 \times \theta}{360°} = 6$$

$$\Rightarrow \quad \theta = \frac{6 \times 360°}{2\pi \times 5.2} \qquad \textbf{1}$$

Now, area of sector $= \dfrac{\theta}{360°} \times \pi r^2$

$$= \frac{6 \times 360°}{2\pi \times 5.2 \times 360°} \times \pi \times (5.2)^2$$

$= 15.6$ sq. units. **1**

AI **Q. 3. The minute hand of a clock is 12 cm long. Find the area of the face of the clock described by the minute hand in 35 minutes.**

A [CBSE Delhi Set-III, 2020]

Sol. $\because$ Angle subtended in 1 minute $= \frac{360°}{60°} = 6°$ ½

$\therefore$ Angle subtended in 35 minutes $= 6° \times 35$

$= 210°$ ½

Area of the face of the clock described by the minute hand in 35 minutes

$=$ Area of the sector (210°)

$= \frac{\pi r^2\theta}{360°}$ ½

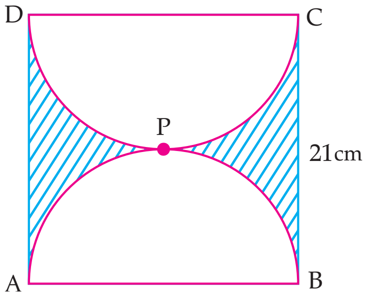
$= \frac{22}{7} \times \frac{12 \times 12 \times 210°}{360°}$

$= \frac{665280}{2520}$

$= 264$ cm^2. ½

Q. 4. Find the perimeter of the shaded region if ABCD is a square of side 21 cm and APB and CPD are semicircles. $\left(\textbf{Use } \pi = \frac{22}{7}\right)$

U [CBSE SQP, 2016]



Sol. Perimeter of the shaded region

$= AD + BC +$ lengths of the arcs of semi circles

APB and *CPD* **1**

$= 21 + 21 + 2\left(\frac{22}{7} \times \frac{21}{2}\right)$ ½

$= 42 + 66$

$= 108$ cm. ½

[CBSE Marking Scheme, 2016]

Detailed Solution:

Given, ABCD is a square, then

$AB = BC = CD = DA$

$\therefore$ Side of the square $= 21$ cm

Since APB and DPC are are two semicircles.

$\therefore$ radius $(r) = \frac{21}{2}$ cm

and length of arc $APB =$ length of arc DPC

$\therefore$ Length of arc $(APB + DPC)$

$= (2\pi r)$

$= 2 \times \frac{22}{7} \times \frac{21}{2}$

$= 22 \times 3$

$= 66$ cm **1**

$\therefore$ Total perimeter ofl the shaded region

$= AD + BC +$ lengths of the arcs $(APB + DPC)$

$= (21 + 21 + 66)$ cm

$= 108$ cm. **1**

Q. 5. Find the area of the square that can be inscribed in a circle of radius 8 cm.

U [CBSE Board Term-2, 2015]

Sol. Diameter of the circle = diagonal of square

$= 2 \times 8 = 16$ cm

Let x be the side of square.

$\therefore$ $x\sqrt{2} = 16$ or, $x = 8\sqrt{2}$ **1**

Area of square $= x^2 = \left(8\sqrt{2}\right)^2$

$= 128$ cm^2 **1**

[CBSE Marking Scheme, 2015]

Detailed Solution:

Radius of the circle $= 8$ cm.

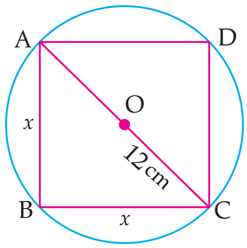
$\therefore$ Diameter of circle $= 16$ cm.

$\therefore$ Diagonal of square $= 16$ cm

Let the side of square $= x$ cm.

$x^2 + x^2 = (16)^2$

(Pythagoras theorem)



1

or, $2x^2 = 16 \times 16$

or, $x^2 = \frac{16 \times 16}{2} = 128$

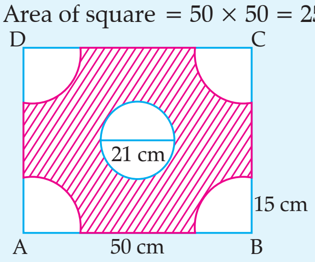
Area of square $= x^2 = 128$ cm^2. **1**

AI **Q. 6. A child prepares a poster on "save water" on a square sheet whose each side measures 50 cm. At each corner of the sheet, she draws a quadrant of radius 15 cm in which she shows the ways to save water. At the centre, she draws a circle of diameter 21 cm and writes a slogan save water in it. Find the area of the remaining sheet.**

C [CBSE Board Term-2, 2015]

Sol. Side of square $= 50$ cm

$\therefore$ Area of square $= 50 \times 50 = 2500$ cm^2 ½



Radius of quadrant $= 15$ cm.

Area of 4 quadrants $= 4 \times \frac{1}{4}\pi r^2 = \pi r^2$

$= \pi \times 15 \times 15$

$= \frac{22}{7} \times 225$

$= 707.14 \text{ cm}^2$ ½

Area of circle $= \pi r^2$

$= \frac{22}{7} \times \left(\frac{21}{2}\right)^2$

$= \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2}$

$= 346.5 \text{ cm}^2$

Area of remaining sheet = Area of square

– 4(area of quadrant)

– Area of circle ½

$= 2500 - 707.14 - 346.5$

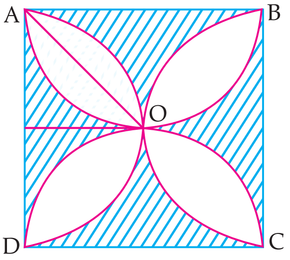
$= 1446.36$ sq. cm **1**

[CBSE Marking Scheme, 2015]

Short Answer Type Questions-II

3 marks each

AI **Q. 1. In the figure, ABCD is a square of side 14 cm. Semi-circles are drawn with each side of square as diameter. Find the area of the shaded region.**



A [CBSE SQP, 2020-21]
[CBSE Delhi Set-I, II, III, 2016]

Sol. Area of 1 segment = area of sector – area of triangle ½

$= \left(\frac{90°}{360°}\right)\pi r^2 - \frac{1}{2} \times 7 \times 7$

$= \frac{1}{4} \times \frac{22}{7} \times 7^2 - \frac{1}{2} \times 7 \times 7$ ½

$= 14 \text{ cm}^2$ ½

Area of 8 segments $= 8 \times 14 = 112 \text{ cm}^2$ ½

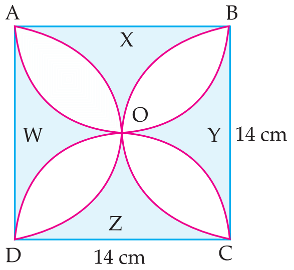
Area of the shaded region $= 14 \times 14 - 112$ ½

$= 196 - 112 = 84 \text{ cm}^2$ ½

(each petal is divided into 2 segments)

[CBSE Marking Scheme, 2020-21]

Detailed Solution:



Side of a square = 14 cm

∴ Area of the square *ABCD* $= 14 \times 14$

$= 196 \text{ cm}^2$

Area of semicircle *AOB* $= \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \frac{22}{7} \times 7 \times 7$

$= 77 \text{ cm}^2$ **1**

Similarly, area of semicircle *DOC* $= 77 \text{ cm}^2$

Hence, the area of shaded region (Part W and Part Y)

= Area of square

– Area of two semicircles *AOB* and *COD*

$= 196 - 154 = 42 \text{ cm}^2$ **1**

Therefore, area of four shaded parts (*i.e.*, X, Y, W, Z)

$= 2 \times 42 = 84 \text{ cm}^2$ **1**

Q. 2. The area of a circular playground is 22176 cm². Find the cost of fencing this ground at the rate of 50 per metre. A [CBSE OD Set-I, 2020]

Sol. Area of a circular play ground $= 22176 \text{ cm}^2$

i.e., $\pi r^2 = 22176 \text{ cm}^2$ **1**

[where r is the radius of a play ground]

$\Rightarrow r^2 = 22176 \times \frac{7}{22} = 7056$

$\Rightarrow r = 84 \text{ cm} = 0.84 \text{ m}$ **1**

Cost of fencing this ground $=$ ₹ $50 \times 2\pi r$

$=$ ₹ $50 \times 2 \times \frac{22}{7} \times 0.84$

$=$ ₹ 264. **1**

Q. 3. Sides of a right triangular field are 25 m, 24 m and 7 m. At the three corners of the field, a cow, a buffalo and a horse are tied separately with ropes of 3.5 m each to graze in the field. Find the area of the field that cannot be grazed by these animals. A [CBSE SQP, 2020]

Sol. Required Area = Area of triangle

– Area of 3 sectors

Area of Triangle $= \frac{1}{2} \times 24 \times 7 = 84 \text{ m}^2$ **1**

Area of three sectors $= \frac{\pi r^2}{360°}$

$\times$ (Sum of three angles of triangle)

$= \frac{22 \times 7 \times 7 \times 180°}{7 \times 2 \times 2 \times 360°}$

$= \frac{77}{4}$ or 19.25 m^2 **1**

∴ Required Area $= (84 - 19.25) \text{ m}^2$

$= 64.75 \text{ m}^2$ **1**

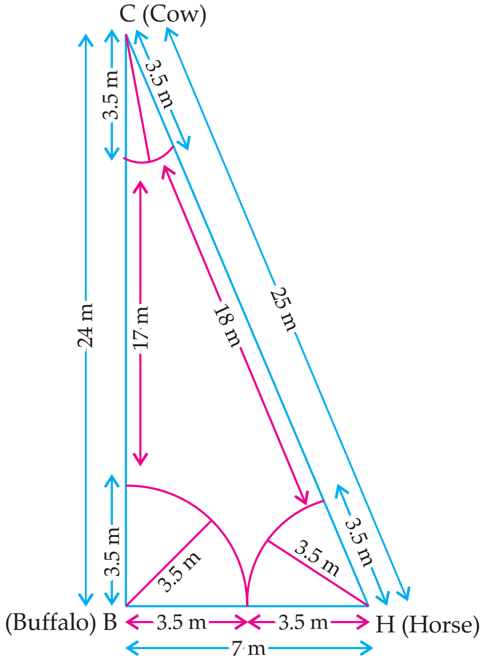
[CBSE SQP Marking Scheme, 2020]

Detailed Solution:

Given that, a triangular field with the three corners of the field a cow, a buffalo and a horse are tied separately with ropes.

So, each animal grazed the field in each corner of triangular field as a sectorial form.

Given that radius of each sector



½

$$(r) = 3.5 \text{ m}$$

$$\text{Area of sector } \angle C = \frac{\angle C}{360^\circ}\pi r^2 = \frac{\angle C}{360^\circ}\pi(3.5)^2 \text{ m}^2$$

$$\text{Area of sector } \angle B = \frac{\angle B}{360^\circ}\pi r^2 = \frac{\angle B}{360^\circ}\pi(3.5)^2 \text{ m}^2$$

$$\text{Area of sector } \angle H = \frac{\angle H}{360^\circ}\pi r^2 = \frac{\angle H}{360^\circ}\pi(3.5)^2 \text{ m}^2$$

Sum of the areas of three sectors ½

$$= \frac{\angle C}{360^\circ}\pi(3.5)^2 + \frac{\angle B}{360^\circ}\pi(3.5)^2 + \frac{\angle H}{360^\circ}\pi(3.5)^2$$

$$= \left(\frac{\angle C + \angle B + \angle H}{360^\circ}\right) \times \frac{22}{7} \times 3.5 \times 3.5$$

$[\because \angle C + \angle B + \angle H = 180^\circ]$

$$= \frac{180^\circ}{360^\circ} \times 38.5$$

$$= 19.25 \text{ m}^2$$ ½

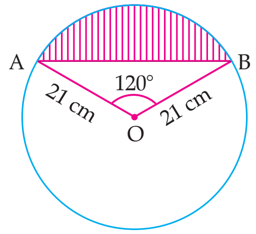
Since ΔCBH is a right angled triangle,

then $$\text{area of } \Delta CBH = \frac{1}{2} \times CB \times BH = \frac{1}{2} \times 24 \times 7 = 84 \text{ m}^2$$ ½

So, area of the field which cannot be grazed by three animals = Area of ΔCBH – Area of three sectors

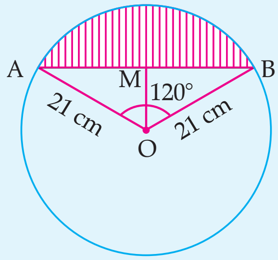
$$= (84 - 19.25) \text{ m}^2 = 64.75 \text{ m}^2.$$ 1

Q. 4. Find the area of the segment shown in Figure, if radius of the circle is 21 cm and $\angle AOB = 120^\circ$. $\left(\text{Use } \pi = \frac{22}{7}\right)$



A **[CBSE Delhi Set-II, 2019]**

Sol. Draw MO ⊥ AB



$$\angle OAB = \angle OBA = 30^\circ$$ ½

$$\sin 30^\circ = \frac{1}{2} = \frac{OM}{21} \Rightarrow OM = \frac{21}{2}$$

$$\cos 30^\circ = \frac{\sqrt{3}}{2} = \frac{AM}{21} \Rightarrow AM = \frac{21}{2}\sqrt{3}$$

$$\text{Area of } \Delta OAB = \frac{1}{2} \times AB \times OM = \frac{1}{2} \times 21\sqrt{3} \times \frac{21}{2} = \frac{441}{4}\sqrt{3} \text{ cm}^2.$$ 1

∴ Area of shaded region

= Area (sector $OAYB$) – Area (ΔOAB)

$$= \frac{22}{7} \times 21 \times 21 \times \frac{120}{360} - \frac{441}{4}\sqrt{3}$$ 1

$$= \left(462 - 441\frac{\sqrt{3}}{4}\right) \text{ cm}^2$$

$$= 271.3 \text{ cm}^2 \text{ (approx.)}$$ ½

[CBSE Marking Scheme, 2019]

Detailed Solution:

Given that, radius of a circle $(r) = 21$ cm

and in ΔAOB, $OA = OB$

$\therefore \angle OAB = \angle OBA$

(Isosceles property)

So, $2\angle OAB = 180° - 120°$

(Angle sum property)

$\Rightarrow \angle OAB = \frac{60°}{2} = 30°$

$\therefore \angle OAB = \angle OBA = 30°$

In ΔOMB, $\sin 30° = \frac{OM}{OB}$

$\Rightarrow \frac{1}{2} = \frac{MB}{21}$

$\Rightarrow OM = \frac{21}{2}$ ½

and $\cos 30° = \frac{MV}{OB}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{MB}{OB}$

$\Rightarrow MB = \frac{21\sqrt{3}}{2}$ ½

$\therefore AB = 2 \times MB$

$= 2 \times \frac{21\sqrt{3}}{2}$

$= 21\sqrt{3}$ cm

Area of $\Delta OAB = \frac{1}{2} \times AB \times OM$

$= \frac{1}{2} \times 21\sqrt{3} \times \frac{21}{2}$

$= \frac{441\sqrt{3}}{4}$ cm^2 ½

Area of sector $OACB = \frac{\theta}{360°} \times \pi r^2$

$= \frac{120°}{360°} \times \frac{22}{7} \times 21 \times 21$

$= 462$ cm^2 1

Hence, area of shaded region

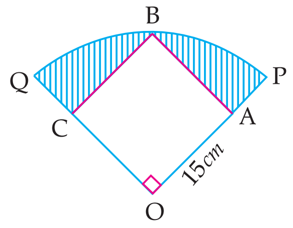
= area of sector $OACB$

– area of ΔOAB

$= \left(462 - \frac{441 \times 1.73}{4}\right)$ cm^2

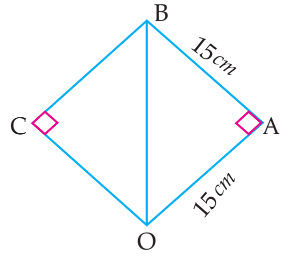
$= 271.3$ cm^2. ½

Q. 5. In Figure, a square OABC is inscribed in a quadrant OPBQ. If *OA* = 15 cm, find the area of the shaded region. (Use π = 3.14)



A **[CBSE OD Set-I, 2019]**

Sol. Since OABC is a square.



Then, $\angle OAB = 90°$

$\therefore$ In ΔOAB,

Radius of quadrant $= OB = \sqrt{15^2 + 15^2}$

(By Pythagoras theorem)

$\Rightarrow OB = 15\sqrt{2}$ cm **1**

Now, area of quadrant $OQBP$

$= \frac{\theta}{360°} \times \pi r^2$

$= \frac{90°}{360°} \times 3.14 \times (OB)^2$

$= \frac{1}{4} \times 3.14 \times (15\sqrt{2})^2$

$= 353.25$ cm^2 ½

Area of square $= (OA)^2 = (15)^2$

$= 225$ cm^2 ½

Hence,

area of shaded region = area of quadrant

– area of square

$= (353.25 - 225)$ cm^2

$= 128.25$ cm^2 **1**

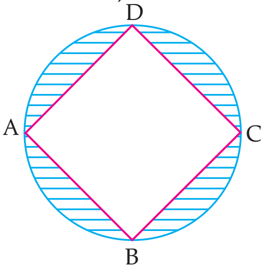
COMMONLY MADE ERROR

- Many candidates use incorrect formula for finding the area of shaded region.

ANSWERING TIP

- Remember all the formulae related to square and quadrant of the circle.

Q. 6. In figure, ABCD is a square with side $2\sqrt{2}$ cm and inscribed in a circle. Find the area of the shaded region. (Use π = 3.14)



A [CBSE OD Set-I, 2019]

Sol. $BD = \sqrt{(2\sqrt{2})^2 + (2\sqrt{2})^2} = \sqrt{16} = 4$ cm **1**

$\therefore$ Radius of circle = 2 cm ½

$\therefore$ Shaded area = Area of circle – Area of square ½

$= 3.14 \times 2^2 - (2\sqrt{2})^2$

$= 12.56 - 8 = 4.56$ cm^2 **1**

[CBSE Marking Scheme, 2019]

Detailed Solution:

Given, side of square, $a = 2\sqrt{2}$ cm.

$\therefore$ area of square $= a^2 = (2\sqrt{2})^2 = 8$ cm^2

½

We know that, the length of the diagonal of a square is given by, $d = a\sqrt{2}$

$\Rightarrow$ $d = 2\sqrt{2} \times \sqrt{2} = 4$ cm ½

Since, the square is inscribed in a circle, hence the diagonal of square will be the diameter of the circle,

i.e., radius, $r = \frac{d}{2} = \frac{4}{2} = 2$ cm ½

$\therefore$ Area of the circle $= \pi r^2$

$= 3.14 \times (2)^2$

$= 12.56$ cm^2 ½

Therefore, area of shaded region

= Area of circle – Area of the square

= (12.56 – 8) = 4.56 cm^2 **1**

COMMONLY MADE ERROR

- Some candidates either use incorrect formula or made errors is calculation.

ANSWERING TIP

- Understand all the formula related to area of circle.

Q. 7. A wiper blade has length 21 cm, sweeps 120°. Calculate the area swept by two blades.

[CBSE Delhi Region, 2019]

Topper's Answer, 2019

Sol. given, each wiper blade has length (r) = 21cm.
and sweeps angle = 120°

Area swept by oneblade = $\frac{\theta}{360} \times \pi r^2$ unit square

$= \frac{120}{360} \times \frac{22}{7} \times 21 \times 21$ cm^2

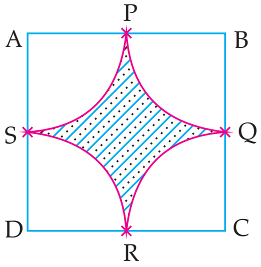
$= \underline{462\text{ cm}^2}$

Blades donot overlap.

$\therefore$ Area swept by 2blades = 462×2 cm^2 = $\underline{924\text{ cm}^2}$

3

Q. 8. Find the area of the shaded region in given figure, where arcs drawn with centres A, B, C and D intersect in pairs at mid-points P, Q, R and S of the sides AB, BC, CD and DA respectively of a square ABCD of side 12 cm. [Use π = 3.14] A [CBSE Delhi/OD, 2018]



Sol. Radius of each arc drawn $= 6$ cm ½

Area of one quadrant $= (3.14) \times \dfrac{36}{4}$

Area of four quadrants $= 3.14 \times 36$ **1**

$= 113.04 \text{ cm}^2$ **1**

Area of square $ABCD = 12 \times 12 = 144 \text{ cm}^2$

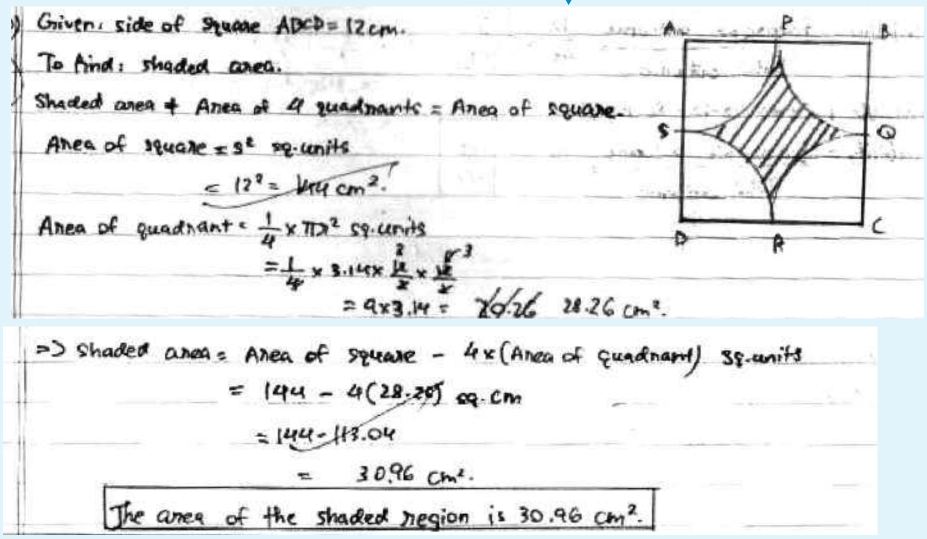
Hence, Area of shaded region $= 144 - 113.04$

$= 30.96 \text{ cm}^2$ ½

[CBSE Marking Scheme, 2018]

Detailed Solution:

Topper's Answer, 2018



Given: side of square ABCD = 12 cm.

To Find: shaded area.

Shaded area + Area of 4 quadrants = Area of square.

Area of square = s^2 sq.units

= 12^2 = 144 cm².

Area of quadrant = $\frac{1}{4} \times \pi r^2$ sq.units

= $\frac{1}{4} \times 3.14 \times \frac{12}{2} \times \frac{12}{2}$

= 9×3.14 = 28.26 cm².

⇒ Shaded area = Area of square − 4×(Area of quadrant) sq.units

= 144 − 4(28.26) sq.cm

= 144 − 113.04

= 30.96 cm².

The area of the shaded region is 30.96 cm². **3**

Q. 9. The short and long hands of a clock are 4 cm and 6 cm long respectively. Find the sum of distances travelled by their tips in 48 hours.

C + A [CBSE Compt Set-I, II, III, 2018]
[Foreign Set-I, II, III, 2015]

Sol. Distance travelled by short hand in 48 hours

$= 4 \times 2\pi \times 4$ cm $= 32\pi$ cm. **1**

Distance travelled by long hand in 48 hours

$= 48 \times 2\pi \times 6$ cm $= 576\pi$ cm **1**

Total distance travelled $= (32\pi + 576\pi)$ cm

$= 608\pi$ cm **1**

[CBSE Marking Scheme, 2018]

Detailed Solution:

Short hand makes 4 rounds in 48 hours

Long hand makes 48 rounds in 48 hours ½

Radius of the circle formed by short hand = 4 cm

and radius of the circle formed by long hand = 6 cm

Distance travelled by short hand in one round

= circumference of the circle

$= 2 \times 4 \times \pi = 8\pi$ cm ½

Distance travelled by short hand in 4 rounds

$= 2 \times 4 \times 4\pi = 32\pi$ cm ½

Distance travelled by long hand in one round

$= 2 \times \pi \times 6 = 12\pi$ cm ½

Distance travelled by long hand in 48 rounds

$= 48 \times 12\pi = 576\pi$ cm ½

Sum of the distances $= 32\pi + 576\pi = 608\pi$ cm ½

Q. 10. The side of a square is 10 cm. Find the area between inscribed and circumscribed circles of the square.

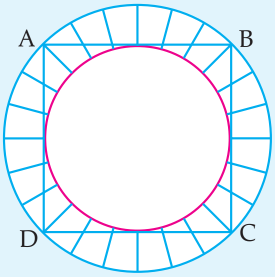
A [CBSE Comptt. Set-I, II, III, 2018]

Sol. Radius of inner circle = 5 cm ½

Radius of outer circle $= 5\sqrt{2}$ cm **1**

Required area = Area of outer circle

− Area of inner circle **1**



$= \pi[(5\sqrt{2})^2 - 5^2] = 25\pi \text{ cm}^2$ ½

[CBSE Marking Scheme, 2018]

Detailed Solution:

Here, diameter of inner circle = side of the square = 10 cm

$\therefore$ Radius of inner circle $= \frac{10}{2} = 5$ cm **1**

Diameter of outer circle = Diagonal of square $= 10\sqrt{2}$ cm

$\therefore$ Radius of outer circle $= \frac{10\sqrt{2}}{2} = 5\sqrt{2}$ cm **1**

Then, the required Area = Area of outer circle – Area of inner circle

$= \pi[(5\sqrt{2})^2 - (5)^2]$

$= 25\pi$ cm^2 **1**

Q. 11. A wire, when bent in the form of an equilateral triangle, encloses an area of $121\sqrt{3}$ cm^2. If the wire is bent in the form of a circle, find the area enclosed by the circle, $\left(\text{Use } \pi = \frac{22}{7}\right)$

A [CBSE OD Set-I, II, III, 2017]

Sol. Let a be the side of triangle.

$\therefore$ Area enclosed by the triangle $= \frac{\sqrt{3}}{4}a^2$

$\Rightarrow \quad \frac{\sqrt{3}}{4}a^2 = 121\sqrt{3}$

$\Rightarrow \quad a^2 = \frac{121\sqrt{3} \times 4}{\sqrt{3}}$

$a = 22$ cm ½

Perimeter of triangle = circumference of circle formed.

$\therefore \quad 2\pi r = 22 \times 3$ **1**

$\Rightarrow \quad 2 \times \frac{22}{7} \times r = 22 \times 3$

$\Rightarrow \quad r = \frac{22 \times 3 \times 7}{22 \times 2} = \frac{21}{2}$ cm ½

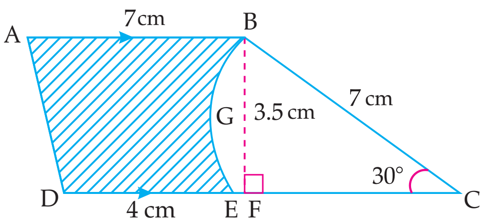
Area enclosed by the circle $= \pi r^2$

$= \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} = \frac{693}{2}$

$= 346.5$ cm^2 (Approx) **1**

[CBSE Marking Scheme, 2017]

Q. 12. In adjoining fig, ABCD is a trapezium with $AB \parallel DC$ and $\angle BCD = 30°$. Fig. BGEC is a sector of a circle with centre C and $AB = BC = 7$ cm, $DE = 4$ cm and $BF = 3.5$ cm, then find the the area of the shaded region. $\left(\text{Use } \pi = \frac{22}{7}\right)$



A [CBSE OD Comptt. Set-I, II, III, 2017]

Sol. Given, $AB = 7$ cm, $DE = 4$ cm, and $BF = 3.5$ cm

$DC = DE + EC = 4 + 7 = 11$ cm

Area of trapezium ABCD

$= \frac{1}{2}$ (Sum of || lines) × (distance between them)

$= \frac{1}{2}(11+7) \times 3.5$

$= \frac{1}{2} \times 18 \times 3.5$

$= 31.5$ cm^2 **1**

Area of the sector BGEC

$= \frac{30°}{360°} \times \frac{22}{7} \times 7 \times 7 = \frac{1}{12} \times 22 \times 7 = 12.83$ cm^2 (Approx.) **1**

Area of shaded region

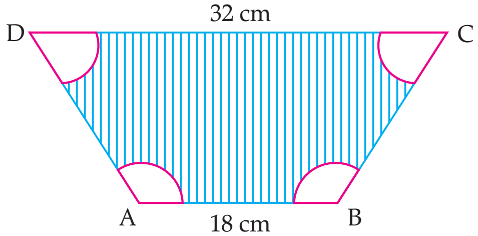
= Area of trapezium – Area of sector

$= 31.5 - 12.83$

$= 18.67$ cm^2 (Approx.) **1**

[CBSE Marking Scheme, 2017]

Q. 13. In the given figure ABCD is a trapezium with $AB \parallel DC$, $AB = 18$ cm and $DC = 32$ cm and the distance between AB and DC is 14 cm. If arcs of equal radii 7 cm taking A, B, C and D have been drawn, then find the area of the shaded region.



A [Foreign Set-I, II, III, 2017]

Sol. Given, in trapezium ABCD, $AB = 18$ cm, $CD = 32$ cm, $AB \parallel CD$ and distance between || lines = 14 cm and the radius of each sector = 7 cm.

Area of trapezium $ABCD = \frac{1}{2}(18+32) \times 14$

$= \frac{1}{2} \times 50 \times 14$

$= 350$ cm^2 **1**

Let, $\angle A = \theta_1$, $\angle B = \theta_2$, $\angle C = \theta_3$ and $\angle D = \theta_4$

ar of sector $A = \frac{\theta_1}{360°}\pi r^2$

$= \frac{\theta_1}{360°} \times \frac{22}{7} \times 7 \times 7$

$= \frac{\theta_1}{360°} \times 154$ cm^2

ar of sector $B = \frac{\theta_2}{360°} \times 154$ cm^2

ar of sector $C = \frac{\theta_3}{360°} \times 154$ cm^2

ar of sector $D = \frac{\theta_4}{360°} \times 154$

ar of 4 sectors $= \frac{\theta_1 + \theta_2 + \theta_3 + \theta_4}{360°} \times 154$

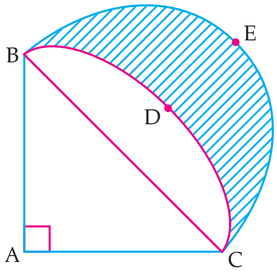
$$= \frac{360°}{360°} \times 154$$

$$= 154 \text{ cm}^2 \quad \mathbf{1}$$

$\therefore$ Area of shaded region = 350 – 154 = 196 cm^2 **1**

[CBSE Marking Scheme, 2017]

Q. 14. ABDC is a quadrant of a circle of radius 28 cm and a semi-circle BEC is drawn with BC as diameter. Find the area of the shaded region. $\left(\text{Use } \pi = \frac{22}{7}\right)$



A **[CBSE SQP, 2017]**

Sol. As ABC is a quadrant of the circle, ∠BAC will be measured 90°.

In ΔABC, $BC^2 = AC^2 + AB^2$

$$= (28)^2 + (28)^2$$

$$= 2(28)^2$$

$\therefore$ $BC = 28\sqrt{2}$ cm

Radius of semi-circle drawn on $BC = \frac{28\sqrt{2}}{2}$

$$= 14\sqrt{2} \text{ cm} \quad \tfrac{1}{2}$$

Area of semi-circle $= \frac{1}{2} \times \frac{22}{7} \times (14\sqrt{2})^2$

$$= 616 \text{ cm}^2 \quad \mathbf{1}$$

Area of $\Delta ABC = \frac{1}{2} \times 28 \times 28$

$$= 392 \text{ cm}^2 \quad \tfrac{1}{2}$$

Area of of quadrant $= \frac{1}{4} \times \frac{22}{7} \times 28 \times 28$

$$= 616 \text{ cm}^2 \quad \mathbf{1}$$

Area of the shaded region

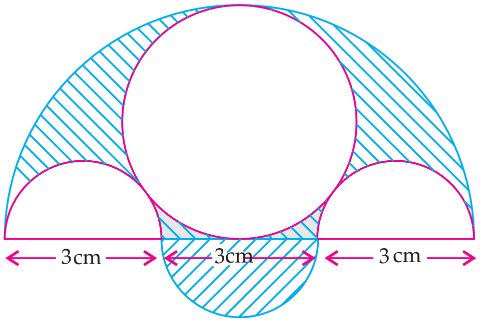
= Area of semi-circle + Area of Δ – Area of quadrant

= 616 + 392 – 616

= 392 cm^2. ½

[CBSE Marking Scheme, 2017]

Q. 15. Three semicircles each of diameter are 3 cm, a circle of diameter 4.5 cm and a semicircle of radius 4.5 cm are drawn in the given figure. Find the area of the shaded region.



A **[CBSE OD Set II, 2017]**

Topper Answer, 2017

Sol. Area of shaded region = Area of semicircle with r = 4.5 cm
+ Area of semicircle with d = 3cm
– 2X area of semicircle with d = 3 cm
– area of circle with d = 4.5cm.

$$= \frac{1}{2} \times \pi \times (4.5)^2 + \left(\frac{1}{2} \times \pi \times \left(\frac{3}{2}\right)^2\right) - 2 \times \left(\frac{1}{2} \times \pi \left(\frac{3}{2}\right)^2\right) - \pi \left(\frac{4.5}{2}\right)^2$$

$$= \frac{2 \times 1}{} \times \pi \times \frac{20.25}{} - \pi \times \frac{9}{} - \pi \times \frac{20.25}{}$$

$$= \frac{\pi}{4}\left[\frac{2 \times 20.25}{2} - \frac{9}{2} - 20.25\right]$$

$= \frac{\pi}{4} [40.5 - 4.5 - 20.25]$

$= \pi/4 [20.25 - 4.5]$

$= \frac{\pi}{4} (15.75)$

$= \frac{22}{7 \times 4} \times 15.75$

$= \frac{11 \times 2.25}{2}$

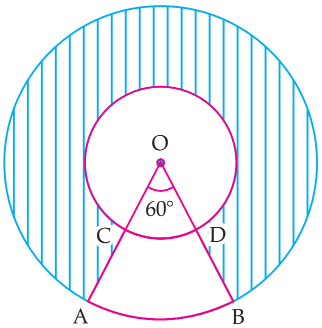
$= \frac{24.75}{2}$

$= 12.375 \text{ cm}^2$

$\therefore$ area of shaded region is 12.375 cm^2

3

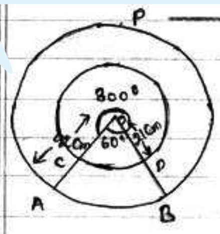
Q. 16. In the given figure, two concentric circles with centre O have radii 21 cm and 42 cm. If $\angle AOB = 60°$, find the area of the shaded region. $\left(\text{Use } \pi = \frac{22}{7}\right)$



A [CBSE OD Set III, 2017]
[CBSE OD, Set-I, II, III, 2016]

Topper Answer, 2017

Sol.



$\angle AOB = \angle COD = 60°$ $R = 42 \text{ cm}, r = 21 \text{ cm}$

$\therefore$ reflex of $\angle AOB = 300° = \theta \; (360° - 60°)$

Now,

area of shaded region

$= \frac{\theta}{360°} \times \pi R^2 - \frac{\theta}{360°} \times \pi r^2$

$= \frac{\theta}{360°} \times \pi \times (R^2 - r^2)$

$= \frac{300°}{360°} \times \frac{22}{7} \times (42-21)(42+21)$

$= \frac{5}{6} \times \frac{22}{7} \times 21 \times 63$

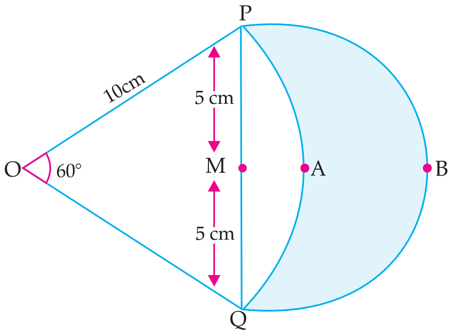
$= 5 \times 11 \times 63$

$= 3465 \text{ cm}^2$

area of shaded region is 3465 cm^2 or 0.3465 m^2

3

Q. 17. Figure shows two arcs PAQ and PBQ. Arc PAQ is a part of circle with centre O and radius OP while arc PBQ is a semi-circle drawn on PQ as diameter with centre M. If $OP = PQ = 10$ cm, show that area of shaded region is $25\left(\sqrt{3}-\frac{\pi}{6}\right)\text{cm}^2$.



A [CBSE Delhi Set I, II, III, 2016]

Sol. Given, $OP = OQ = PQ = 10$ cm

and $\angle POQ = 60°$ ½

Area of segment PAQM

$$= \left(\frac{100\pi}{6} - \frac{100\sqrt{3}}{4}\right)\text{cm}^2. \quad 1$$

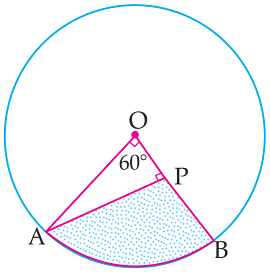
Area of semicircle $= \frac{25\pi}{2}$ cm^2 ½

$$\text{Area of shaded region} = \frac{25\pi}{2} - \left(\frac{50\pi}{3} - 25\sqrt{3}\right).$$

$$= 25\left(\sqrt{3}-\frac{\pi}{6}\right)\text{cm}^2. \quad 1$$

[CBSE Marking Scheme, 2016]

Q. 18. In the given figure, AOB is a sector of angle 60° of a circle with centre O and radius 17 cm. If $AP \perp OB$ and $AP = 15$ cm, find the area of the shaded region.



A [CBSE S.A.2, 2016]

Sol. As $OA = 17$ cm, $AP = 15$ cm and ΔOPA is right triangle.

$\therefore$ Using Pythagoras theorem in ΔOPA,

$$OP = \sqrt{OA^2 - AP^2}$$

$$= \sqrt{(17)^2 - (15)^2}$$

$\Rightarrow$ $OP = 8$ cm 1

Area of the shaded region

= Area of the sector $AOBA$

– Area of ΔOPA 1

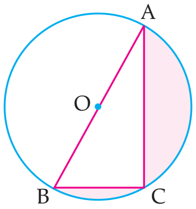
$$= \frac{60°}{360°}\times\pi r^2 - \frac{1}{2}\times b\times h$$

$$= \frac{60°}{360°}\times\frac{22}{7}\times 17\times 17 - \frac{1}{2}\times 8\times 15$$

$= 151.38 - 60$

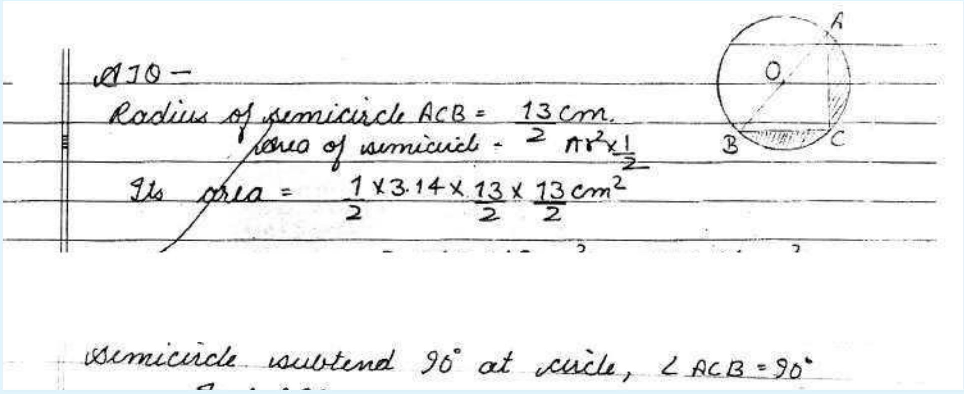
$= 91.38$ cm^2 1

Q. 19. In the given figure, O is the centre of circle such that diameter $AB = 13$ cm and $AC = 12$ cm. BC is joined. Find the area of the shaded region. ($\pi = 3.14$) A [CBSE OD Set-I, II, III, 2016]



Topper's Answer, 2016

Sol.



Q10 —

Radius of semicircle ACB = 13/2 cm.

Area of semicircle = $\pi r^2 \times \frac{1}{2}$

Its area = $\frac{1}{2} \times 3.14 \times \frac{13}{2} \times \frac{13}{2}$ cm^2

Semicircle subtend 90° at circle, ∠ACB = 90°

In Δ ABC –

$AC^2 + BC^2 = AB^2 \Rightarrow 12^2 + BC^2 = 169 \text{ cm}^2$

$\Rightarrow BC^2 = (169 - 144) \text{ cm}^2 \quad BC^2 = 25 \text{ cm}$

$BC = 5 \text{ cm}.$

Area of Δ = $\frac{1}{2}$ × Base × Height

Area of Δ ABC = $\frac{1}{2} \times AC \times BC = \frac{1}{2} \times 12 \times 5 = 30 \text{ cm}^2$

Area of shaded region = $\frac{530.66}{8} \text{ cm}^2 - 30 \text{ cm}^2$

$\Rightarrow (66.3325 - 30) \text{ cm}^2$

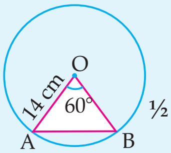
$\Rightarrow$ 36.3325 cm^2 **3**

Q. 20. Find the area of minor segment of a circle of radius 14 cm, when its central angle is 60°. Also, find the area of corresponding major segment.

(Use $\pi = \frac{22}{7}$) A [CBSE OD Set I, II, II, 2015]

Sol. Here, r = 14 cm, θ = 60°

Then, the area of minor segment



$$= \pi r^2 \frac{\theta}{360^\circ} - \frac{1}{2} r^2 \sin\theta \quad ½$$

$$= \frac{22}{7} \times 14 \times 14 \times \frac{60^\circ}{360^\circ} - \frac{1}{2} \times 14 \times 14 \times \frac{\sqrt{3}}{2} \quad ½$$

$$= (\frac{308}{3} - 49\sqrt{3}) \text{ cm}^2 = 17.89 \text{ cm}^2 = 17.9 \text{ approx.} \quad 1$$

and the area of major segment $= \frac{22}{7} \times 14 \times 14$

$$- (\frac{308}{3} - 49\sqrt{3})$$

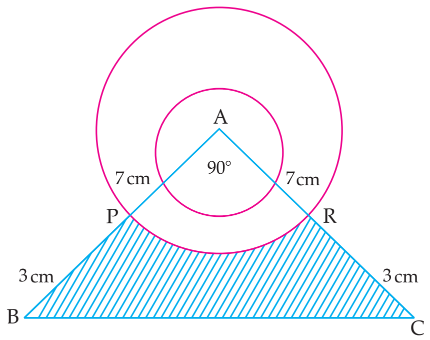
$$= \frac{1540}{3} + 49\sqrt{3} = 598.10 \text{ cm}^2$$

$$= 598 \text{ cm}^2 \text{ approx.} \quad 1$$

[CBSE Marking Scheme, 2015]

Q. 21. A momento is made as shown in the figure. Its base PBCR is silver plated from the front side. Find the area which is silver plated.

(Use $\pi = \frac{22}{7}$)



A [CBSE Term-2, 2015]

Sol. From the given figure

Area of right-angled ΔABC = $\frac{1}{2} \times 10 \times 10$

$= 50 \text{ cm}^2$ **1**

Area of quadrant APR of the circle of radius 7 cm

$= \frac{1}{4} \times \pi \times (7)^2$

[∵ Area of quadrant $= \frac{90^\circ}{360^\circ} \pi r^2$]

$= \frac{1}{4} \times \frac{22}{7} \times 49$

$= 38.5 \text{ cm}^2$ **1**

∴ Area of base $PBCR$ = Area of ΔABC

– Area of quadrant APR

$= 50 - 38.5 = 11.5 \text{ cm}^2$. **1**

[CBSE Marking Scheme, 2015]

Q. 22. The circumference of a circle exceeds the diameter by 16.8 cm. Find the radius of the circle.

(Use $\pi = \frac{22}{7}$) U [CBSE Term-2, 2015]

Sol. Let radius of the circle be r cm.

Diameter = $2r$ cm

Circumference = $2\pi r$ ½

Circumference = Diameter + 16.8 ½

or, $2\pi r = 2r + 16.8$

or, $2\left(\frac{22}{7}\right) r = 2r + 16.8$

or, $\frac{44}{7} r = 2r + 16.8$

or, $44r = 14r + 16.8 \times 7$

or, $30r = 117.6$

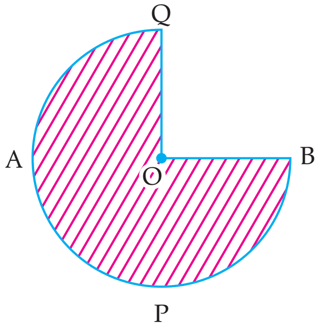
or, $r = \frac{117.6}{30}$ **1**

∴ $r = 3.92$ cm **1**

[CBSE Marking Scheme, 2015]

Q. 23. In fig., APB and AQP are semi-circles, and $AO = OB$. If the perimeter of the figure is 47 cm, find the area of the shaded region. $\left(\text{Use } \pi = \frac{22}{7}\right)$

A [CBSE Delhi Set-I, II, III, 2015]



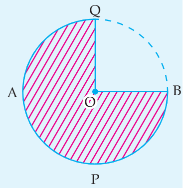
Sol. Let 'r' be the radius of given circle.

Perimeter of given figure = 47 cm

Perimeter of full circle – perimeter of $\left(\frac{1}{4}\right)^{th}$ circle

$$= 47 - 2r \quad \frac{1}{2}$$

or, $$2\pi r - \frac{1}{4}(2\pi r) = 47 - 2r \quad \frac{1}{2}$$



or, $$\frac{3\pi r}{2} + 2r = 47 \text{ cm}$$

or, $$r\left(\frac{3}{2}\times\frac{22}{7}+2\right) = 47 \text{ cm}$$

or, $$r\left(\frac{33}{7}+2\right) = 47 \text{ cm}$$

or, $$r = \frac{47\times 7}{47} \text{ cm}$$

or, $$r = 7 \text{ cm} \quad 1$$

Now, area of shaded region

$$A = \text{area of circle} - \frac{1}{4}\text{area of circle}$$

$$= \frac{3}{4}\text{ area of circle}$$

$$= \frac{3}{4}\times \pi r^2$$

$$= \frac{3}{4}\times\frac{22}{7}\times 7\times 7 \text{ cm}^2$$

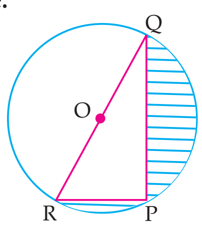
$$= \frac{3}{2}\times 77 \text{ cm}^2$$

$$= 115.5 \text{ cm}^2 \quad 1$$

[CBSE Marking Scheme, 2015]

Long Answer Type Questions — 5 marks each

AI **Q. 1. Find the area of the shaded region in the given figure if $PQ = 24$ cm, $PR = 7$ cm and O is the centre of the circle.**



A [CBSE OD Set-I, 2020]

Sol. Given, $PQ = 24$ cm, $PR = 7$ cm

We know that the angle in the semicircle is right angle.

Here, $\angle RPQ = 90°$ 1

In ΔRPQ,

$$RQ^2 = PR^2 + PQ^2$$

(By Pythagoras theorem)

$\Rightarrow$ $$RQ^2 = (7)^2 + (24)^2 = 49 + 576 = 625$$

$\therefore$ $$RQ = 25 \text{ cm} \quad 1$$

$\therefore$ Area of $\Delta RPQ = \frac{1}{2} \times RP \times PQ$

$$= \frac{1}{2}\times 7\times 24 = 84 \text{ cm}^2 \quad 1$$

and area of semi-circle $= \frac{1}{2}\times \pi r^2$

$$= \frac{1}{2}\times\frac{22}{7}\times\left(\frac{25}{2}\right)^2$$

$$= \frac{11\times 625}{7\times 4} = \frac{6875}{28} \text{ cm} \quad 1$$

Now, area of shaded region

$= \text{area of semi-circle} - \text{area of } \Delta RPQ$

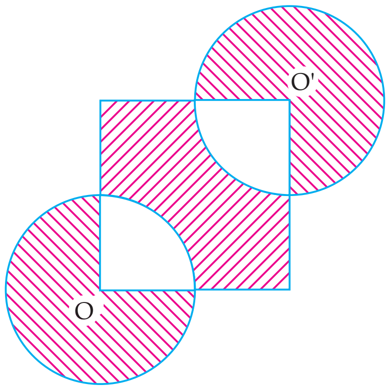
$$= \frac{6875}{28} - 84 = \frac{6875-2352}{28}$$

$$= \frac{4523}{28}$$

$$= 161.54 \text{ cm}^2. \quad 1$$

Q. 2. In the given figure, the side of square is 28 cm and radius of each circle is half of the length of the side of the square, where O and O′ are centres of the circle. Find the area of shaded region.

C + A [CBSE Delhi Set-I, II, 2017]



Sol. Given, the side of the square = 28 cm.

Area of the square = 28 × 28 = 784 cm **1**

Radius of each circle $= \frac{28}{2} = 14$ cm **1**

∴ Area of two circles $= 2 \times \frac{22}{7} \times 14 \times 14$

$= 1232 \text{ cm}^2$ **1**

Area of the 2 quadrants $= \frac{90^\circ}{360^\circ} \times \pi \times 14 \times 14 \times 2$

$= 308 \text{ cm}^2$ **1**

Area the shaded region = Area of square

+ Area of two circles

– area of two quadrants

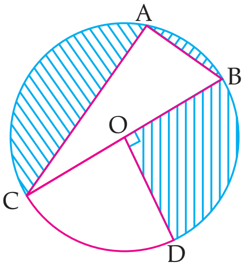
= 784 + 1232 – 308

$= 1708 \text{ cm}^2$

Hence, the area of shaded region $= 1708 \text{ cm}^2$ **1**

[CBSE Marking Scheme, 2017]

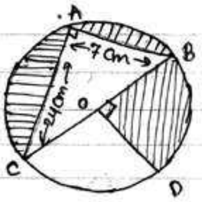
Q. 3. In the given figure, O is the centre of the circle with *AC* = 24 cm, *AB* = 7 cm and ∠ *BOD* = 90°. Find the area of the shaded region.



C + A **[CBSE OD Set-III, 2017]**

Topper Answer, 2017

Sol.



∠CAB = 90° – angle subtended by diameter.

in right ΔCAB,

by pythagoras theorem,

$AC^2 + AB^2 = BC^2$

$24^2 + 7^2 = BC^2$

$576 + 49 = BC^2$

$625 = BC^2$ —(ignoring -ve value)

∴ BC = 25 cm. – diameter.

∴ radius = 12.5 cm or ~~25/2 cm.~~

∴ area of shaded region = area of semicircle + area of quadrant – area of Δ ACB

$= \frac{1}{2} \times \pi r^2 + \frac{1}{4} \times \pi r^2 - \frac{1}{2} \times AB \times AC$

$= \frac{3}{4} \pi r^2 - \frac{1}{2} \times 7 \times 24$

$= \frac{3}{4} \times \frac{22}{7} \times \frac{625}{4} - 7 \times 12$

= 368.3035 – 84

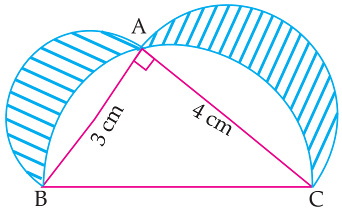
= 284.3035

≈ 284.3 cm²

∴ The area of shaded region is 284.3035 cm²

Q. 4. In the given figure, ΔABC is a right angled triangle in which ∠*A* = 90°. Semicircles are drawn on AB, AC and BC as diameters. Find the area of the shaded region.

C + A **[CBSE OD Set-II, 2017]**



Sol. In ΔABC, $\angle A = 90°$, $AB = 3$ cm, and $AC = 4$ cm

$\therefore$ By using Pythagoras theorem,

$$BC = \sqrt{AB^2 + AC^2} = \sqrt{3^2 + 4^2}$$

$$= 5 \text{ cm.} \qquad 1$$

Area of semicircle with radius $\frac{3}{2}$ cm + Area of semi-circle with radius $\frac{4}{2}$ cm $= \frac{\pi}{2}\left(\frac{3}{2}\right)^2 + \frac{\pi}{2}\left(\frac{4}{2}\right)^2$ ½

Area of semicircle with radius $\frac{5}{2}$ cm – Area of

$$\Delta ABC = \frac{\pi}{2}\left(\frac{5}{2}\right)^2 - \frac{1}{2}\times 3\times 4$$

$$= \left(\frac{25}{8}\pi - 6\right)\text{cm}^2 \qquad \text{...(i) } 1$$

Area of shaded region

$$= \frac{\pi}{2}\left(\frac{3}{2}\right)^2 + \frac{\pi}{2}(2)^2 - \left[\frac{25}{8}\pi - 6\right]\text{cm}^2 \qquad 1$$

$$= \frac{\pi}{2}\left[\frac{9}{4} + 4 - \frac{25}{4}\right] + 6$$

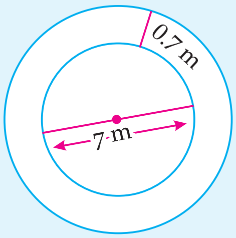
$$= \frac{\pi}{2}\left[\frac{9}{4} + \frac{16-25}{4}\right] + 6$$

$$= \frac{\pi}{2}\left[\frac{9}{4} - \frac{9}{4}\right] + 6 \qquad 1$$

$= 6$ cm^2 **[CBSE Marking Scheme, 2017]** ½

Q. 5. A park is of the shape of a circle of diameter 7 m. It is surrounded by a path of width of 0.7 m. Find the expenditure of cementing the path. If its cost is ₹ 110 per sq. m. C + A **[Foreign Set-I, II, III, 2017]**

Sol.



Given, the diameter of park = 7 m ½

$$\therefore \quad \text{Radius} = \frac{7}{2} = 3.5 \text{ m} \qquad 1$$

The width of path = 0.7 m

$\therefore$ Radius of park with path

= 3.5 + 0.7 = 4.2 m 1

$$\text{Area of the path} = \pi(4.2)^2 - \pi(3.5)^2$$

$$= \frac{22}{7}(17.64 - 12.25) \qquad ½$$

$$= \frac{22}{7}\times 5.39 = 22 \times 0.77$$

$$= 16.94 \text{ m}^2 \qquad 1$$

Cost of the cementing the path

= 16.94 × 110

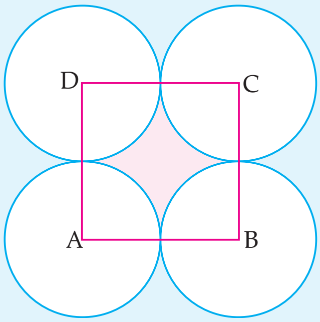
= ₹ 1863.40 1

[CBSE Marking Scheme, 2017]

Q. 6. Four equal circles are described at the four corners of a square so that each touches two others. The shaded area enclosed between the circles is $\frac{24}{7}$ cm^2. Find the radius of each circle.

A **[CBSE SQP, 2016]**

Sol.



1

Let r cm be the radius of each circle.

$$\text{Area of square} - \text{Area of 4 sectors} = \frac{24}{7} \text{ cm}^2 \qquad 1$$

$$\text{or,} \quad (2r)^2 - 4\left(\frac{90°}{360°}\times \pi r^2\right) = \frac{24}{7} \qquad 1$$

$$\text{or,} \quad 4r^2 - \frac{22}{7}r^2 = \frac{24}{7} \qquad 1$$

$$\text{or,} \quad \frac{28r^2 - 22r^2}{7} = \frac{24}{7}$$

or, $6r^2 = 24$

or, $r^2 = 4$ 1

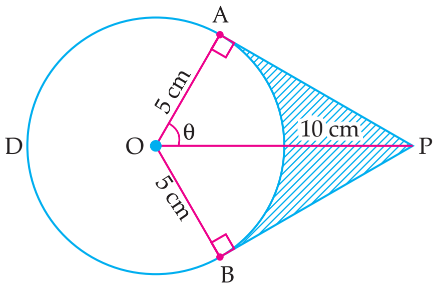
or, $r = \pm 2$

Radius of each circle is 2 cm (r cannot be negative)

[CBSE Marking Scheme, 2016]

Q. 7. An elastic belt is placed around the rim of a pulley of radius 5 cm. From one point C on the belt, elastic belt is pulled directly away from the centre O of the pulley until it is at P, 10 cm from the point O. Find the length of the belt that is still in contact with the pulley. Also find the shaded area.

(Use π = 3.14 and $\sqrt{3}$ = 1.73)



C + A **[CBSE Delhi Set I, II, III, 2016]**

Sol. In right angled ΔOAP,

$$\cos\theta = \frac{5}{10} = \frac{1}{2} \text{ or, } \theta = 60° \qquad 1$$

So, $\angle AOB = 60° + 60° = 120°$

Reflex $\angle AOB = 240°$ 1

$\therefore$ Length of arc

$$ADB = \frac{2\times 3.14\times 5\times 240°}{360°}$$

$$= 20.93 \text{ cm} \qquad 1$$

$$\left[\because l = \frac{\theta}{360^\circ} \times 2\pi r\right]$$

Hence length of elastic in contact = 20.93 cm

$$AP = \sqrt{OP^2 - OA^2}$$

[By using Pythagoras theorem]

$$= \sqrt{(10)^2 - (5)^2} = \sqrt{75}$$

Now, $AP = 5\sqrt{3}$ cm

Area ($\Delta OAP + \Delta OBP$) = Area of ΔOAP + Area of ΔOBP

$$= 2 \times \frac{1}{2} \times 5 \times 5\sqrt{3}$$

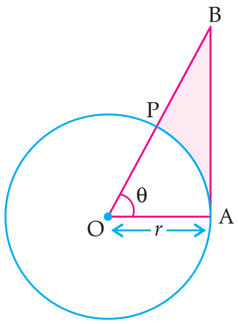
$$= 25\sqrt{3} = 43.25 \text{ cm}^2$$ ½

Area of sector $OACB = \dfrac{25 \times 3.14 \times 120^\circ}{360^\circ}$

$$= 26.16 \text{ cm}^2$$ ½

Shaded Area = 43.25 – 26.16 = 17.09 cm² 1

Q. 8. The given fig. shows a sector OAP of a circle with centre O, containing ∠θ. AB is perpendicular to the radius OA and meets OP produced at B. Prove that the perimeter of shaded region is $r\left[\tan\theta + \sec\theta + \dfrac{\pi\theta}{180^\circ} - 1\right]$

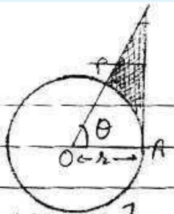


C + A [CBSE OD Set-I,II, III, 2015, 16]

Topper Answer, 2016

Sol.

Given – OAP is sector of circle with centre O, ∠POA = θ and OA⊥AB



To prove –

Perimeter of shaded region = $r\left[\tan\theta + \sec\theta + \frac{\pi\theta}{180} - 1\right]$

Proof –

Perimeter of shaded region = BP + AB + arc $\widehat{AP}$ — (IV)

Now –

$\tan\theta = \dfrac{AB}{r} \Rightarrow r\tan\theta = AB$ — (1)

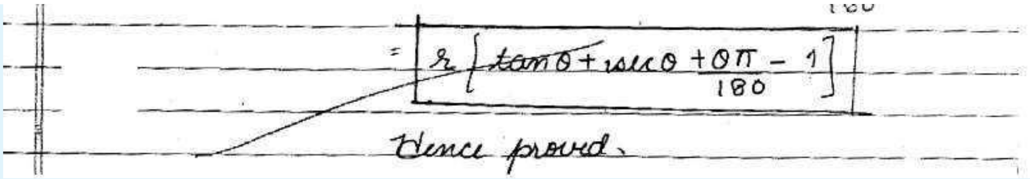
$\sec\theta = \dfrac{OB}{r} \Rightarrow r\sec\theta = OB$

$OB - OP = BP \Rightarrow r\sec\theta - r = BP$ — (2)

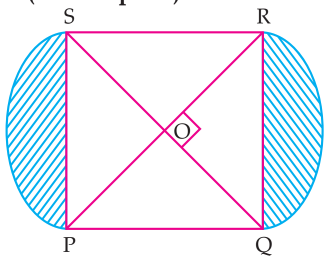
Length of arc AP = $\dfrac{\theta}{360} \times 2\pi r = \dfrac{\theta \times 2\pi r}{360} = \dfrac{\theta\pi r}{180}$ — (3)

Putting value from eq (1), (2) (3) in eq IV

$= r\tan\theta + r\sec\theta - r + \dfrac{\theta\pi r}{180}$



Q. 9. In figure, PQRS is square lawn with side PQ = 42 metre. Two circular flower beds are there on the sides PS and QR with centre at O, the intersection of its diagonals. Find the total area of the two flower beds (shaded parts).



C + A **[CBSE OD Set-I, II, III, 2015]**

Sol. Radius of circle with centre O is OR.

Let $OR = x$

$\therefore \quad x^2 + x^2 = (42)^2$ or, $x = 21\sqrt{2}$ m **1**

(Using pythagoras theorem)

Area of the flower bed = Area of segment of circle with centre angle 90°

$$= \frac{22}{7}\times 21\sqrt{2}\times 21\sqrt{2}\times\frac{90°}{360°}$$

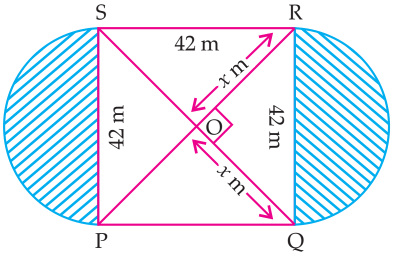
$$-\frac{1}{2}\times 21\sqrt{2}\times 21\sqrt{2}$$ **1**

$= 693 - 441 = 252$ m^2 **2**

$\therefore$ Area of flower beds $= 2 \times 252 = 504$ m^2. **1**

[CBSE Marking Scheme, 2015]

Detailed Solution:



1

Let OR be x cm, then $OQ = x$ cm

Since, diagonals of a square bisect each other at right angles,

then $\angle QOR = 90°$

In ΔQOR, $x^2 + x^2 = (42)^2$

[Using Pythagoras theorem]

$\Rightarrow \quad 2x^2 = 1764$

$\Rightarrow \quad x^2 = 882$

$\Rightarrow \quad x = 21\sqrt{2}$ m **1**

i.e., radius (r) of each semi-circle $= 21\sqrt{2}$ m

Now, area of sector $OQR = \frac{90°}{360°}\times\pi r^2$

$= \frac{1}{4}\times\frac{22}{7}\times 882$ m^2

$= 693$ m^2 **1**

And area of right angled ΔQOR

$= \frac{1}{2}\times OQ\times OR$

$= \frac{1}{2}\times 21\sqrt{2}\times 21\sqrt{2}$

$= 441$ m^2 ½

Area of flower bed (shaded part) with QR

= area of sector OQR

– area of ΔQOR

$= (693 - 441)$ m^2 $= 252$ m^2 ½

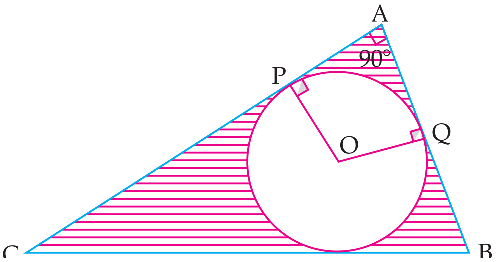
Similarly, area of flower bed with $PS = 252$ m^2

Hence Total area of both flower beds with QR and PS

$= (252 + 252)$ m^2

$= 504$ m^2. **1**

Q. 10. In the figure, ABC is a right angled triangle right angled at ∠A. Find the area of the shaded region, if AB = 6 cm, BC = 10 cm and O is the centre of the incircle of the triangle ABC.



C + A **[CBSE Term-2, 2015]**

Sol. Let r be the radius of incircle.

In ΔBAC,

$AC = \sqrt{BC^2 - AB^2}$

(By using Pythagoras theorem)

$= \sqrt{10^2 - 6^2}$

$= \sqrt{100 - 36}$

$= \sqrt{64}$

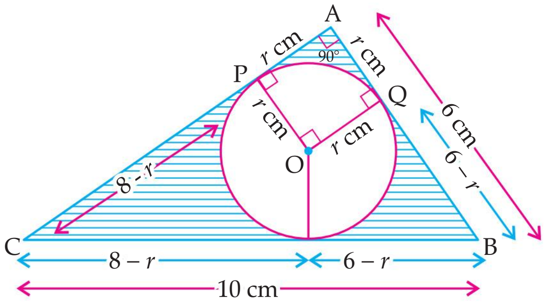
$= 8$ cm **1**

Here, $\angle A = 90°$, $\angle P = 90°$ and $\angle Q = 90°$

Then $\angle O = 360° - (90° + 90° + 90°)$

$= 90°$

∴ OPAQ is a square.

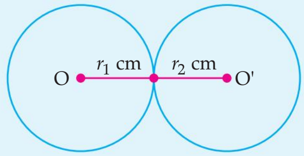


Then,	$BC = 10 = 8 - r + 6 - r$	**1**
	(By using the tangent properties)	
or,	$2r = 8 + 6 - 10$	
or,	$2r = 4$ or, $r = 2$ cm	
∴	Area of circle $= \pi r^2 = \frac{22}{7} \times 2 \times 2$	
	$= \frac{88}{7} = 12.57$ cm^2	**1**
Now,	area of $\Delta ABC = \frac{1}{2} \times 8 \times 6$	
	$= 24$ cm^2	**1**
∴	Area of shaded region = Area of ΔABC – Area of circle	
	$= 24 - 12.57$	
	$= 11.43$ cm^2	**1**

Q. 11. Two circular beads of different sizes are joined together such that the distance between their centres is 14 cm. The sum of their areas is 130 π cm^2. Find the radius each bead.

C + A [CBSE Term-2, 2015]

Sol. Let the radii of the circles are r_1 cm and r_2 cm.



∴	$r_1 + r_2 = 14$	(Given) ...(i)	
and sum of their areas	$= \pi r_1^2 + \pi r_2^2$		
	$130\pi = \pi(r_1^2 + r_2^2)$	(Given)	**1**
or,	$130\pi = \pi(r_1^2 + r_2^2)$		
∴	$r_1^2 + r_2^2 = 130$	...(ii)	**1**
	$(r_1 + r_2)^2 = r_1^2 + r_2^2 + 2r_1r_2$		
or,	$(14)^2 = 130 + 2r_1r_2$		
or,	$2r_1r_2 = 196 - 130$		
	$= 66$		**1**
	$(r_1 - r_2)^2 = r_1^2 + r_2^2 - 2r_1r_2$		
	$= 130 - 66$		
	$= 64$		
or,	$r_1 - r_2 = 8$	...(iii)	**1**
From (i) and (iii),	$2r_1 = 22$		
or,	$r_1 = 11$ cm		
and	$r_2 = 14 - 11$		
	$= 3$ cm.		**1**

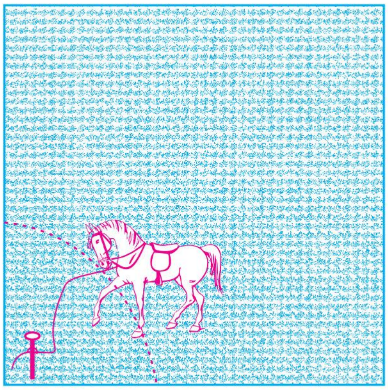
[CBSE Marking Scheme, 2015]

Visual Case Based Questions

4 marks each

Note: Attempt any four sub parts from each question. Each sub part carries 1 mark

AI **Q. 1. A horse is tied to a peg at one corner of a square shaped grass field of sides 15 m by means of a 5 m long rope (see the given figure).** C + AE



(i) What is the area of the grass field?

(a) **225 m^2** **(b)** **225 m**

(c) **255 m^2** **(d)** **15 m**

Sol. Correct option: **(a)**

Explanation: Area of square = (side)^2

$= 15 \times 15$

$= 225$ m^2 **1**

(ii) The area of that part of the field in which the horse can graze.

(a) **19.625 m^2** **(b)** **19.265 m^2**

(c) **19 m^2** **(d)** **78.5 m^2**

Sol. Correct option: **(a)**

Explanation: From the figure, it can be observe that the horse can graze a sector of 90° in a circle of 5 m radius.

Area that can be grazed by horse

= Area of sector

$= \frac{90°}{360°} \times \pi r^2$

$= \frac{1}{4} \times 3.14 \times 5 \times 5$

$= 19.625$ m^2 **1**

(iii) The grazing area if the rope were 10 m long instead of 5 m.

(a) 7.85 m^2 **(b)** 785 m^2

(c) 225 m^2 **(d)** 78.5 m^2

Sol. Correct option: **(d)**

Explanation: Area that can be grazed by the horse when length of rope is 10 m long

$$= \frac{90°}{360°} \times \pi r^2$$

$$= \frac{1}{4} \times 3.14 \times 10 \times 10$$

$$= 78.5 \text{ m}^2$$ 1

(iv) The increase in the grazing area if the rope were 10 m long instead of 5 m.

(a) 58.758 m^2 **(b)** 58.875 m^2

(c) 58 m^2 **(d)** 78.5 m^2

Sol. Correct option: **(b)**

Explanation: Increase in grazing area

$$= (78.5 - 19.625) \text{ m}^2$$

$$= 58.875 \text{ m}^2$$ 1

(v) The given problem is based on which concept?

(a) Coordinate geometry

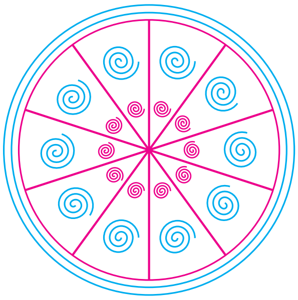
(b) Area related to circles

(c) Circle

(d) None of these

Sol. Correct option: **(b)** 1

AI **Q. 2. In a workshop, brooch is made with silver wire in the form of a circle with diameter 35 mm. The wire is also used in making 5 diameters which divide the circle into 10 equal sectors as shown in the given figure.** C + AE



(i) What is the radius of the circle?

(a) $\frac{35}{2}$ **mm** **(b)** $\frac{5}{2}$ **mm**

(c) 35 mm **(d) 10 mm**

Sol. Correct option: **(a)**

Explanation: Radius of circle $= \frac{\text{Diameter}}{2}$

$$= \frac{35}{2} \text{ mm}$$ 1

(ii) What is the circumference of the brooch?

(a) 100 mm **(b) 110 mm**

(c) 50 mm **(d) 10 mm**

Sol. Correct option: **(b)**

Explanation: Circumference of brooch $= 2\pi r$

$$= 2 \times \frac{22}{7} \times \frac{35}{2}$$

$$= 110 \text{ mm}$$ 1

(iii) What is the total length of silver wire required?

(a) 528 mm **(b) 825 mm**

(c) 285 mm **(d) 852 mm**

Sol. Correct option: **(c)**

Explanation:

Length of wire required $= 110 + 5 \times 35$

$= 110 + 175$

$= 285$ mm. 1

(iv) What is the area of the each sector of the brooch?

(a) $\frac{385}{2}$ **mm²** **(b)** $\frac{358}{4}$ **mm²**

(c) $\frac{585}{4}$ **mm²** **(d)** $\frac{385}{4}$ **mm²**

Sol. Correct option: **(d)**

Explanation: It can be observed from the figure that an angle of each 10 sectors of the circle is subtending at the centre of the circle.

$$\therefore \text{ Area of each sector } = \frac{36}{360°} \times \pi r^2$$

$$= \frac{1}{10} \times \frac{22}{7} \times \frac{35}{2} \times \frac{35}{2}$$

$$= \frac{385}{4} \text{ mm}^2$$ 1

(v) The given problem is based on which mathematical concept ?

(a) Areas Related to circles

(b) Circles

(c) Construction

(d) none of these

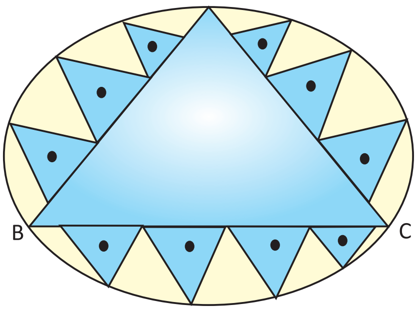
Sol. Correct option: **(a).** 1

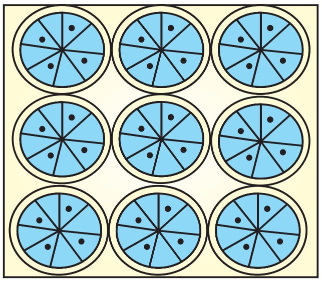
Q. 3. AREAS RELATED TO CIRCLES

Pookalam is the flower bed or flower pattern designed during Onam in Kerala. It is similar as Rangoli in North India and Kolam in Tamil Nadu.

During the festival of Onam , your school is planning to conduct a Pookalam competition. Your friend who is a partner in competition , suggests two designs given below.

Observe these carefully.





Design I: This design is made with a circle of radius 32 cm leaving equilateral triangle ABC in the middle as shown in the given figure.

Design II: This Pookalam is made with 9 circular design each of radius 7 cm.

Refer Design I:

(i) The side of equilateral triangle is

(a) $12\sqrt{3}$ cm (b) $32\sqrt{3}$ cm

(c) 48 cm (d) 64 cm

Sol. Correct option: **(b).**

(ii) The altitude of the equilateral triangle is

(a) 8 cm (b) 12 cm

(c) 48 cm (d) 52 cm

Sol. Correct option: **(c).**

Refer Design II:

(iii) The area of square is

(a) 1264 cm^2 (b) 1764 cm^2

(c) 1830 cm^2 (d) 1944 cm^2

Sol. Correct option: **(b).**

Explanation:

radius = 7 cm

diameter = 2 × 7 cm

= 14 cm

side of square = 14 cm + 14 cm + 14 cm

= 42 cm.

Area of square = side^2

= $(42 \text{ cm})^2$

= 1764 cm^2

(iv) Area of each circular design is

(a) 124 cm^2 (b) 132 cm^2

(c) 144 cm^2 (d) 154 cm^2

Sol. Correct option: **(d).**

Explanation:

radius = 7 cm

Area of each circular design

$= \pi r^2$

$= \frac{22}{7} \times 7 \times 7$

= 154 cm^2

(v) Area of the remaining portion of the square ABCD is

(a) 378 cm^2 (b) 260 cm^2

(c) 340 cm^2 (d) 278 cm^2

Sol. Correct option: **(a).**

Explanation:

Area of 9 circular design

$= 9 \times \pi r^2$

$= 9 \times \frac{22}{7} \times 7 \times 7$

= 1386 cm^2

Area of square = 1764 cm^2

Area of remaining portion of square ABCD

= Area of square – Area of 9 circular design

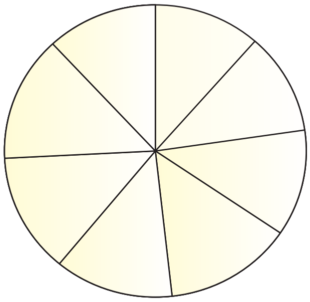
= 1764 cm^2 – 1386 cm^2

= 378 cm^2

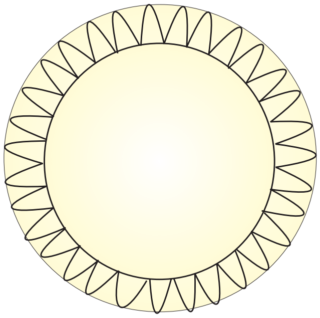
Q. 4. A Brooch

A brooch is a small piece of jewellery which has a pin at the back so it can be fastened on a dress, blouse or coat.

Designs of some brooch are shown below. Observe them carefully.



A



B

Design A: Brooch A is made with silver wire in the form of a circle with diameter 28 mm. The wire used for making 4 diameters which divide the circle into 8 equal parts.

Design B: Brooch b is made two colours *i.e.* Gold and silver. Outer part is made with Gold. The circumference of silver part is 44 mm and the gold part is 3 mm wide everywhere.

Refer to Design A

(i) The total length of silver wire required is

(a) 180 mm **(b) 200 mm**

(c) 250 mm **(d) 280 mm**

Sol. Correct option: **(b).**

Explanation:

$$\begin{aligned} \text{Diameter} &= 28 \text{ mm} \\ \text{radius} &= 14 \text{ mm} \\ \text{Total length of wire} &= \text{length of 4 diameter} \\ &\quad + \text{circumference of circle.} \\ &= 4 \times 28 + 2\pi r^2 \\ &= 112 + 2 \times \frac{22}{7} \times 14 \\ &= 112 + 88 \\ &= 200 \text{ mm} \end{aligned}$$

(ii) The area of each sector of the brooch is

(a) 44 mm^2 **(b) 52 mm^2**

(c) 77 mm^2 **(d) 68 mm^2**

Sol. Correct option: **(c).**

Explanation:

Area of each sector of Broof

$$\begin{aligned} &= \frac{1}{8} \times \text{Area of Brooch} \\ &= \frac{1}{8} \times \pi r^2 \\ &= \frac{1}{8} \times \frac{22}{7} \times 14 \times 14 \\ &= 77 \text{ mm}^2 \end{aligned}$$

Refer to Design B

(iii) The circumference of outer part (golden) is

(a) 48.49 mm **(b) 82.2 mm**

(c) 72.50 mm **(d) 62.86 mm**

Sol. Correct option: **(d).**

(iv) The difference of areas of golden and silver parts is

(a) 18π **(b) 44π**

(c) 51π **(d) 64π**

Sol. Correct option: **(c).**

(v) A boy is playing with brooch B. He makes revolution with it along its edge. How many complete revolutions must it take to cover 80 mm ?

(a) 2 **(b) 3**

(c) 4 **(d) 5**

Sol. Correct option: **(c).**

Explanation:

Circumference of silver part of Brooch

$$\begin{aligned} &= 44 \text{ cm} \\ 2\pi r &= 44 \text{ mm} \\ 2 \times \frac{22}{7} \times r &= 44 \\ r &= 7 \text{ mm.} \end{aligned}$$

radius of whole Brooch

$$\begin{aligned} &= 7\text{mm} + 8 \text{ mm} \\ &= 10 \text{ mm.} \end{aligned}$$

Circumference of outer edge

$$\begin{aligned} &= 2\pi r \\ &= 2 \times \frac{22}{7} \times 10 \end{aligned}$$

$$= \frac{440}{7} \text{ mm}$$

let the number of revolutions = n

Now, According to question,

$$n.2\pi r = 80\pi$$

$$n.\frac{440}{7} = 80\pi$$

$$n.\frac{440}{7} = 80 \times \frac{22}{7}$$

$$n = 4$$